UMD-PP-03-008

Five Dimensional Supergravity in $\mathcal{N}=1$ Superspace

W. D. Linch III,[*] Markus A. Luty,[†] J. Phillips[‡]

Department of Physics, University of Maryland
College Park, Maryland 20742, USA

Abstract

We give a formulation of linearized minimal 5-dimensional supergravity in $\mathcal{N}=1$ superspace. Infinitesimal local 5D diffeomorphisms, local 5D Lorentz transformations, and local 5D supersymmetry are all realized as off-shell superfield transformations. Compactification on an S^1/Z_2 orbifold and couplings to brane-localized supermultiplets are very simple in this formalism. We use this to show that 5-dimensional supergravity can naturally generate μ and $B\mu$ terms of the correct size in gaugino- or radion-mediated supersymmetry breaking. We also include a self-contained review of linearized minimal 4D supergravity in superspace.

[*] ldw@physics.umd.edu

[†] mluty@physics.umd.edu

[‡] ferrigno@physics.umd.edu

1 Introduction

Supergravity is naturally the messenger of supersymmetry breaking in models where supersymmetry is broken in a hidden sector that couples to the visible sector only through gravitational interactions [1]. Supergravity couples universally in the infrared, but theoretical expectations (based on black holes and string theory) are that the fundamental theory of gravity does not respect global symmetries. In the low-energy effective theory, we therefore expect flavor-violating contact terms between the hidden and observable sectors with gravitational strength. These couplings give flavor-dependent contributions to supersymmetry breaking in the visible sector that are generally the same size as the universal contributions from gravity in the infrared, leading to unacceptably large flavor-changing neutral currents.

This problem can be solved in 'brane world' scenarios where the visible and hidden sector are localized on spatially separated 3-branes [2, 3]. In these scenarios, the short-distance behavior of gravity does not affect the transmission of supersymmetry breaking since it must propagate over long distances [3]. Supersymmetry breaking is therefore dominated by infrared gravity, which couples universally and hence can give flavor-independent supersymmetry breaking masses, solving the supersymmetric flavor problem. This leads to a variety of models in which supergravity plays an important role in supersymmetry breaking [3, 4].

The simplest brane-world models are 5-dimensional. Models involving 5D supergravity have been analyzed in a number of ways. Work has been done using the on-shell formulation of 5D supergravity [5], effective field theory techniques [6], and the off-shell formulation of 5D supergravity [7] pioneered by Zucker [8] and further developed by a number of authors [9]. In this paper, we formulate 5D linearized supergravity completely in terms of $\mathcal{N} = 1$ superfields. This approach has recently been developed for global supersymmetry in Ref. [10] (see [11] for earlier work). This means that the fields depend on 4D superspace coordinates (x^m, θ_α) and a 5^{th} coordinate x^5. In this formalism, the full 5D Lorentz invariance and supersymmetry is not manifest, but the advantage is that coupling to 4D matter localized on branes is simple. Some partial results on 5D supergravity using $\mathcal{N} = 1$ superfields have been obtained in Ref. [12].

In this paper, we give the complete linearized action for 5D supergravity in terms of $\mathcal{N} = 1$ superfields. Although global 5D Lorentz invariance and supersymmetry are not manifest, the full infinitesimal 5D local Lorentz transformations, local 5D diffeomorphisms, and local 5D supersymmetry transformations are realized off-shell as superfield transformations. The induced 4D supergravity multiplet on the branes is the standard minimal $\mathcal{N} = 1$ multiplet, and so the couplings to brane-localized matter is simple. As an application of this formalism, we present an operator that gives rise to realistic μ and $B\mu$ terms in the context of gaugino-mediated supersymmetry breaking [13] or radion-mediated supersymmetry breaking (third paper in Ref. [4]).

This paper is organized as follows. Section 2 contains a review of 4D linearized supergravity in superspace, including component results and invariant couplings to matter fields. Section 3 contains the main results of this paper. We show how the 5D supergravity multiplet is embedded in $\mathcal{N} = 1$ superfields, and give the superfield action. We also show that the bosonic terms correctly reproduce 5D linearized gravity along with kinetic terms for the 5D graviphoton. Section 4 applies these results to an S^1/Z_2 orbifold and section 5 gives our conclusions.

2 4D Supergravity in Superspace

In this section we review linearized 4D supergravity in superspace. This formalism is due to Siegel and Gates [14], and is reviewed in Refs. [15, 16].[1]

2.1 Gauge Transformations and Superfield Content

The construction is closely analogous to the construction of $\mathcal{N} = 1$ supersymmetric gauge theory in superspace. We briefly review this construction for the case of a $U(1)$ gauge theory. First, we demand that the gauge transformations take chiral superfields into chiral superfields. This restricts the gauge transformations to have the form

$$\delta\Phi = i\Omega\Phi. \tag{2.1}$$

where Ω is chiral. Next, we note that antichiral superfields do not naturally transform under this restricted gauge group, so that for example $\Phi^\dagger\Phi$ is not gauge invariant. We therefore introduce a gauge connection superfield V and define covariant complex conjugation by

$$\Phi^\ddagger \equiv (1 + V + \mathcal{O}(V^2))\Phi^\dagger. \tag{2.2}$$

We want $\Phi^\ddagger$ to transform in the complex conjugate representation with gauge parameter Ω:

$$\delta\Phi^\ddagger = -i\Omega\Phi^\ddagger. \tag{2.3}$$

[1]We use the spinor conventions of Ref. [17]. We use bispinor notation $\partial_{\alpha\dot{\alpha}} = \sigma^m_{\alpha\dot{\alpha}}\partial_m$, *etc.*

This requires that V transforms as

$$\delta V = -i(\Omega - \Omega^\dagger). \tag{2.4}$$

We now know the superfield content and transformation laws, and can work out the action and components.

We follow similar steps in the construction of linearized supergravity. The starting point is the group of infinitesimal super-diffeomorphisms, acting on a general superfield Ψ as

$$\delta\Psi = \Lambda^\alpha D_\alpha \Psi + \Lambda_{\dot\alpha} \bar{D}^{\dot\alpha} \Psi + \Lambda^m \partial_m \Psi. \tag{2.5}$$

Here $\Lambda_{\dot\alpha} \neq (\Lambda_\alpha)^\dagger$ *a priori*. The use of the differential operators D_α and $\bar{D}_{\dot\alpha}$ rather than the ordinary derivatives $\partial/\partial\theta^\alpha$ and $\partial/\partial\bar\theta^{\dot\alpha}$ is not essential, but it makes it easy to keep global supersymmetry manifest. The full group of super-diffeomorphisms Eq. (2.5) is too large to give a minimal formulation of supergravity. We therefore restrict to the subgroup of diffeomorphisms that takes chiral superfields to chiral superfields. This gives the constraints

$$\bar{D}_{\dot\alpha} \Lambda_\alpha = 0, \qquad \bar{D}_{\dot\alpha} \Lambda_{\beta\dot\beta} = -4i\varepsilon_{\dot\alpha\dot\beta} \Lambda_\beta. \tag{2.6}$$

The most general solution can be parameterized by

$$\Lambda_\alpha = -\tfrac{1}{4} \bar{D}^2 L_\alpha, \qquad \Lambda_{\alpha\dot\alpha} = -2i \bar{D}_{\dot\alpha} L_\alpha + \Omega_{\alpha\dot\alpha}, \tag{2.7}$$

where L_α is an general complex superfield, and $\Omega_{\alpha\dot\alpha}$ is chiral. We will restrict attention to the transformations generated by L_α. (It can be checked that these form a closed subgroup.) The transformation of a chiral superfield Φ can then be written

$$\delta\Phi = -\tfrac{1}{4} \bar{D}^2 (L^\alpha D_\alpha \Phi), \qquad \Phi = \text{chiral}. \tag{2.8}$$

The restricted group of super-diffeomorphisms we have found above does not act naturally on antichiral superfields. The constraints imposed by demanding that the general super-diffeomorphisms in Eq. (2.5) preserve antichiral fields are

$$D_\alpha \Lambda_{\dot\alpha} = 0, \tag{2.9}$$

$$D_\alpha \Lambda_{\beta\dot\beta} = -4i\varepsilon_{\alpha\beta} \Lambda_{\dot\beta}. \tag{2.10}$$

Eq. (2.9) is satisfied by choosing

$$\Lambda_{\dot\alpha} = (\Lambda_\alpha)^\dagger = -\tfrac{1}{4} D^2 \bar{L}_{\dot\alpha}, \tag{2.11}$$

but Eq. (2.10) is inconsistent with the chiral constraints Eq. (2.6), and cannot be imposed. Following the construction of gauge theory, we define a covariantly conjugate superfield

$$\Phi^{\ddagger} \equiv (1 - 2iV^m \partial_m)\Phi^{\dagger} \tag{2.12}$$

such that $\Phi^{\ddagger}$ transforms according to the constrained super-diffeomorphisms given by Eqs. (2.7) and (2.11):

$$\delta\Phi^{\ddagger} = -\tfrac{1}{4}(D^2 \bar{L}_{\dot{\alpha}})\bar{D}^{\dot{\alpha}}\Phi^{\ddagger} + i(\bar{D}^{\dot{\alpha}} L^{\alpha})\partial_{\dot{\alpha}\alpha}\Phi^{\ddagger}. \tag{2.13}$$

This requires

$$\delta V_{\alpha\dot{\alpha}} = \bar{D}_{\dot{\alpha}} L_{\alpha} - D_{\alpha}\bar{L}_{\dot{\alpha}}. \tag{2.14}$$

Note that $(\bar{D}_{\dot{\alpha}} L_{\alpha})^{\dagger} = -D_{\alpha}\bar{L}_{\dot{\alpha}}$, so $V_{\alpha\dot{\alpha}}$ is real.

To summarize, a general superfield Ψ transforms according to Eq. (2.5), with Λ_{α}, $\Lambda_{\dot{\alpha}}$, and Λ^m given by Eqs. (2.7) and (2.11). It is sometimes convenient to make a field redefinition

$$\Psi' = (1 + 2iaV^m\partial_m)\Psi, \tag{2.15}$$

where a is a real parameter. The redefined field transforms as

$$\delta\Psi' = -\tfrac{1}{4}(\bar{D}^2 L^{\alpha})D_{\alpha}\Psi' - \tfrac{1}{4}(D^2\bar{L}_{\dot{\alpha}})\bar{D}^{\dot{\alpha}}\Psi' + i\Big[(1-a)\bar{D}^{\dot{\alpha}}L^{\alpha} + aD^{\alpha}\bar{L}^{\dot{\alpha}}\Big]\partial_{\alpha\dot{\alpha}}\Psi'. \tag{2.16}$$

For $a = 0$, this preserves $\bar{D}_{\dot{\alpha}}\Psi' = 0$, for $a = 1$ it preserves $D_{\alpha}\Psi' = 0$, and for $a = \frac{1}{2}$ it preserves $\Psi'^{\dagger} = \Psi'$. In this way we can define covariant versions of real, chiral, and anti-chiral superfields.

2.2 Components and the Chiral Compensator

To better understand the gauge symmetries above, we consider their action on the components of a chiral superfield Φ. We define the component fields by projection

$$\phi = \Phi|, \qquad \psi_{\alpha} = D_{\alpha}\Phi|, \qquad F = -\tfrac{1}{4}D^2\Phi|, \tag{2.17}$$

where '$|$' denotes evaluation at $\theta = 0$. We then find

$$\delta\phi = \xi^m\partial_m\phi + \varepsilon^{\alpha}\psi_{\alpha}, \tag{2.18}$$

$$\delta\psi_{\alpha} = \eta_{\alpha}^m\partial_m\phi + \xi^m\partial_m\psi_{\alpha} + \lambda_{\alpha}{}^{\beta}\psi_{\beta} + 2\varepsilon_{\alpha}F, \tag{2.19}$$

$$\delta F = \kappa^m\partial_m\phi + \tfrac{1}{2}\eta_{\beta}^m\partial_m\psi^{\beta} + \rho^{\alpha}\psi_{\alpha} + \xi^m\partial_m F + \lambda_{\alpha}{}^{\alpha}F, \tag{2.20}$$

where

$$\xi^m = i\tilde{\sigma}^{m\dot{\alpha}\alpha}\bar{D}_{\dot{\alpha}}L_\alpha|, \tag{2.21}$$

$$\varepsilon_\alpha = -\tfrac{1}{4}\bar{D}^2L_\alpha|, \tag{2.22}$$

$$\eta^m_\alpha = i\tilde{\sigma}^{m\dot{\beta}\beta}D_\alpha\bar{D}_{\dot{\beta}}L_\beta|, \tag{2.23}$$

$$\lambda_\alpha{}^\beta = -\tfrac{1}{4}D_\alpha\bar{D}^2L^\beta|, \tag{2.24}$$

$$\kappa^m = -\tfrac{i}{4}\sigma^m_{\alpha\dot{\alpha}}D^2\bar{D}^{\dot{\alpha}}L^\alpha|, \tag{2.25}$$

$$\rho_\alpha = \tfrac{1}{16}D^2\bar{D}^2L_\alpha|. \tag{2.26}$$

Note that the symmetrized generators $\lambda_{(\alpha\beta)}$ generate local Lorentz transformations. The extra gauge symmetry in the trace $\lambda_\alpha{}^\alpha$ generates scale and $U(1)_R$ transformations. We see that gauging super-diffeomorphisms naturally gives rise to *superconformal* supergravity.

To understand the components further, we go to Wess–Zumino gauge. We define the components of the supergravity multiplet as

$$c_m = V_m|, \tag{2.27}$$

$$\chi_{\alpha\beta\dot{\beta}} = D_\alpha V_{\beta\dot{\beta}}|, \tag{2.28}$$

$$a_m = -\tfrac{1}{4}D^2V_m|, \tag{2.29}$$

$$h_{\alpha\dot{\alpha},\beta\dot{\beta}} = -\tfrac{1}{2}[D_\alpha, \bar{D}_{\dot{\alpha}}]V_{\beta\dot{\beta}}|, \tag{2.30}$$

$$\psi^m_\alpha = \tfrac{i}{16}\tilde{\sigma}^{m\dot{\beta}\beta}\bar{D}^2D_\beta V_{\alpha\dot{\beta}}|, \tag{2.31}$$

$$d_m = \tfrac{1}{32}\{D^2, \bar{D}^2\}V_m|. \tag{2.32}$$

These transform as

$$\delta c_m = -\operatorname{Im}(\xi_m), \tag{2.33}$$

$$\delta\chi_{\alpha\beta\dot{\beta}} = \tfrac{i}{2}\sigma_{m\beta\dot{\beta}}\eta^m_\alpha + 2\varepsilon_{\alpha\beta}\bar{\varepsilon}_{\dot{\beta}}, \tag{2.34}$$

$$\delta a_m = \tfrac{i}{2}\kappa_m, \tag{2.35}$$

$$\delta h_{\alpha\dot{\alpha},\beta\dot{\beta}} = \partial_{\alpha\dot{\alpha}}\operatorname{Re}(\xi_{\beta\dot{\beta}}) - 2(\varepsilon_{\dot{\alpha}\dot{\beta}}\lambda_{\alpha\beta} + \varepsilon_{\alpha\beta}\bar{\lambda}_{\dot{\alpha}\dot{\beta}}), \tag{2.36}$$

$$\delta\psi^\alpha_m = \partial_m\varepsilon^\alpha + \tfrac{i}{2}\tilde{\sigma}^{\dot{\alpha}\alpha}_m\bar{\rho}_{\dot{\alpha}}, \tag{2.37}$$

$$\delta d_m = -\tfrac{1}{2}\partial^n\partial_n\operatorname{Im}(\xi_m) + \left[\tfrac{i}{4}(\tilde{\sigma}_m\sigma_n)_{\dot{\alpha}\dot{\beta}}\partial^n\bar{\lambda}^{\dot{\alpha}\dot{\beta}} + \text{h.c.}\right]. \tag{2.38}$$

We use the gauge freedom in $\operatorname{Im}(\xi_m)$, η^m_α, and κ_m to set

$$c_m, \chi_{\alpha\beta\dot{\beta}}, a_m = 0. \tag{2.39}$$

This leaves a residual gauge symmetry with

$$\eta^m_\alpha = 2i\sigma^m_{\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha}. \tag{2.40}$$

In this gauge, $\mathrm{Re}(\xi_m)$ generates local diffeomorphisms and ε generates local supersymmetry transformations on ϕ and ψ. The symmetric part of $\lambda_{\alpha\beta}$ generates local Lorentz transformations, as before.

We still have the 'extra' gauge symmetry generated by $\lambda_\alpha{}^\alpha$ and ρ_α. To fix this, we introduce the conformal compensator Σ. This is a chiral field transforming as

$$\delta\Sigma = -\tfrac{1}{4}\bar D^2 D^\alpha L_\alpha, \tag{2.41}$$

with components defined by

$$\sigma = \Sigma|, \tag{2.42}$$

$$\zeta_\alpha = D_\alpha \Sigma|, \tag{2.43}$$

$$F_\Sigma = -\tfrac{1}{4}D^2\Sigma|. \tag{2.44}$$

These components transform as

$$\delta\sigma = \partial_m \xi^m - \lambda_\alpha{}^\alpha, \tag{2.45}$$

$$\delta\zeta_\alpha = 2\rho_\alpha + \partial_m \eta^m_\alpha, \tag{2.46}$$

$$\delta F_\Sigma = \partial_m \kappa^m. \tag{2.47}$$

We can use the gauge symmetry generated by $\lambda_\alpha{}^\alpha$ and ρ_α to set

$$\sigma = h^m{}_m, \qquad \zeta_\alpha = 0. \tag{2.48}$$

The first condition does not require any further compensating gauge transformation in Wess–Zumino gauge, where ξ^m is real. The second condition requires a compensating gauge transformation

$$\rho_\alpha = -\tfrac{1}{2}\partial_m \eta^m_\alpha = -i\sigma^m_{\alpha\dot\alpha}\partial_m\bar\varepsilon^{\dot\alpha}, \tag{2.49}$$

so that the gravitino transformation law Eq. (2.37) is modified to

$$\delta\psi^m_\alpha = \left[(\sigma^{mn})_\alpha{}^\beta + \tfrac{3}{2}\eta^{mn}\delta_\alpha{}^\beta\right]\partial_n \epsilon_\beta. \tag{2.50}$$

We can redefine the gravitino field to obtain a more conventional transformation law (see Eq. (3.88) below). However, working out the components in terms of the 'unconventional' gravitino field defined here is easier in this approach.

The remaining gauge symmetry in this gauge consists of diffeomorphisms generated by $\mathrm{Re}(\xi^m)$, local supersymmetry transformations generated by ε_α, and local Lorentz transformations generated by $\lambda_{(\alpha\beta)}$. These are precisely the gauge invariances we expect for supergravity. The remaining supergravity component fields

$$h_{mn}, \quad \psi_m^\alpha, \quad d_m, \quad F_\Sigma \tag{2.51}$$

comprise the minimal off-shell $\mathcal{N} = 1$ supergravity multiplet.

2.3 Invariant Couplings

We now construct the couplings of supergravity to chiral matter fields. We first consider a superpotential term

$$\mathcal{L}_F = \int d^2\theta \, W + \text{h.c.}, \tag{2.52}$$

where W is a chiral superfield transforming as $\delta W = -\frac{1}{4}\bar{D}^2(L^\alpha D_\alpha W)$. Under superdiffeomorphisms we have

$$\delta \int d^2\theta W = \int d^4\theta \, L^\alpha D_\alpha W, \tag{2.53}$$

so it is easy to see that adding

$$\Delta\mathcal{L}_F = \int d^2\theta \, \Sigma W + \text{h.c.} \tag{2.54}$$

makes the superpotential term invariant.

We next consider a Kähler term

$$\mathcal{L}_D = \int d^4\theta \, K \tag{2.55}$$

where K is real and transforms like a real superfield (Eq. (2.16) with $a = \frac{1}{2}$). For example, if Φ is a chiral superfield, then the minimal kinetic term must be modified to

$$K = \Phi^\dagger \Phi + i V^m \Phi^\dagger \overleftrightarrow{\partial}_m \Phi, \tag{2.56}$$

where $A\overleftrightarrow{\partial}B = A\partial B - (\partial A)B$. Then

$$\delta \int d^4\theta \, K = \int d^4\theta \Big[\tfrac{1}{4}\bar{D}^2 D^\alpha L_\alpha + \tfrac{i}{2}\partial_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}} L^\alpha + \text{h.c.}\Big] \cdot K. \tag{2.57}$$

This can be cancelled using the identity

$$\delta\Big([D_\alpha, \bar{D}_{\dot{\alpha}}] V^{\dot{\alpha}\alpha}\Big) = -2\bar{D}^2 D^\alpha L_\alpha - 6i\partial_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}} L^\alpha + \text{h.c.} \tag{2.58}$$

We therefore find that adding

$$\Delta\mathcal{L}_D = \int d^4\theta \Big\{ \tfrac{1}{3}(\Sigma + \Sigma^\dagger) + \tfrac{1}{12}[D_\alpha, \bar{D}_{\dot\alpha}]V^{\dot\alpha\alpha} \Big\} \cdot K \tag{2.59}$$

makes the Kähler term invariant.

We now have enough results for 4D supergravity to tackle the 5D case.

3 5D Supergravity

In this section we construct minimal 5D supergravity in $\mathcal{N} = 1$ superspace. On shell this theory contains a metric g_{MN}, a graviphoton B_M, and a gravitino ψ_M, where $M = 0, \ldots, 3; 5$. The on-shell theory was first constructed by Cremmer [18]. Since we will be interested only in the linear theory, the on-shell action is simply the sum of the kinetic terms for the fields above.

3.1 Superfield Embedding

To find the superfields that parameterize the 5D supergravity fields, we collect the bosonic fields and their gauge transformations. The 5-bein fluctuation fields h_{MN} transform as

$$\delta h_{mn} = \partial_m \xi_n + \lambda_{mn}, \tag{3.1}$$

$$\delta h_{5m} = \partial_5 \xi_m + \lambda_{5m}, \tag{3.2}$$

$$\delta h_{m5} = \partial_m \xi_5 - \lambda_{5m}, \tag{3.3}$$

$$\delta h_{55} = \partial_5 \xi_5, \tag{3.4}$$

where we have used the fact that the generators of local Lorentz transformations are antisymmetric $\lambda_{MN} = -\lambda_{NM}$. The graviphoton fields transform as

$$\delta B_m = \partial_m \alpha, \tag{3.5}$$

$$\delta B_5 = \partial_5 \alpha. \tag{3.6}$$

The fields h_{mn} are contained in the superfields V_m and Σ of $\mathcal{N} = 1$ superfield supergravity, as reviewed in the previous section. We now discuss the superfield embedding of the remaining bosonic fields.

When this theory is compactified on an S^1/Z_2 orbifold, the zero modes of h_{55} and B_5 form a chiral 'radion multiplet.' It is therefore natural to parameterize these fields by a chiral superfield

$$T \sim h_{55} + iB_5 + \cdots = \text{chiral} \tag{3.7}$$

transforming as

$$\delta T = \partial_5 \Omega, \tag{3.8}$$

where

$$\Omega \sim \xi_5 + i\alpha + \cdots = \text{chiral}. \tag{3.9}$$

The field h_{m5} transforms as a gauge field with gauge parameter ξ_5. When 5D supergravity is compactified on S^1 this field parameterizes the Kaluza–Klein gauge boson. It is therefore natural to parameterize h_{m5} by a real 'Kaluza–Klein' superfield

$$K \sim \theta\sigma^m\bar{\theta}h_{m5} + \cdots = \text{real}, \tag{3.10}$$

transforming as

$$\delta K = i(\Omega - \Omega^\dagger) - N, \tag{3.11}$$

where

$$N \sim \theta\sigma^m\bar{\theta}\lambda_{5m} = \text{real}. \tag{3.12}$$

Note that the superfield transformation parameterized by N can be used to completely shift away K, just as the local Lorentz transformations λ_{5m} can be used to shift away h_{m5}.

We now turn to h_{5m}. We do *not* embed h_{5m} in a real superfield

$$K' \sim \theta\sigma^m\bar{\theta}h_{5m} + \cdots = \text{real}, \tag{3.13}$$

because ξ_m is embedded in the superfield L_α as

$$L_\alpha \sim i\bar{\theta}^{\dot{\alpha}}\xi_{\alpha\dot{\alpha}} + \cdots, \tag{3.14}$$

and therefore the ξ_m transformation law of h_{5m} would require

$$\delta K' \sim i\theta^\alpha\partial_5 L_\alpha + \cdots. \tag{3.15}$$

The appearance of explicit factors of superspace coordinates in superfield transformations means that manifest global $\mathcal{N} = 1$ supersymmetry is lost. We instead embed h_{5m} and B_m in a spinor superfield[2]

$$\Psi_\alpha \sim \bar{\theta}^{\dot{\alpha}}(B_{\alpha\dot{\alpha}} + ih_{5,\alpha\dot{\alpha}}) + \cdots. \tag{3.16}$$

[2]The embedding of spin $\frac{3}{2}$ fields in superfields of this type was first considered in Ref. [19].

We obtain the correct transformation law for h_{5m} and B_m if we take Ψ_α to transform as

$$\delta\Psi_\alpha = \partial_5 L_\alpha + \tfrac{i}{4} D_\alpha N. \tag{3.17}$$

We now collect the 5D supergravity $\mathcal{N} = 1$ superfields and their complete transformation laws:

$$\delta T = \partial_5 \Omega, \tag{3.18}$$

$$\delta K = i(\Omega - \Omega^\dagger) - N, \tag{3.19}$$

$$\delta\Psi_\alpha = \partial_5 L_\alpha + \tfrac{i}{4} D_\alpha N. \tag{3.20}$$

As already noted above, we can use the N gauge transformation to completely shift away K. Equivalently, the action depends on K only through the combination

$$\hat{\Psi}_\alpha = \Psi_\alpha + \tfrac{i}{4} D_\alpha K, \tag{3.21}$$

which transforms as

$$\delta\hat{\Psi}_\alpha = \partial_5 L_\alpha - \tfrac{1}{4} D_\alpha \Omega. \tag{3.22}$$

3.2 Invariant Action

We now write an invariant lagrangian for the supergravity fields found above. This can be constructed systematically by working order by order in ∂_5. We find that in order to write an invariant action we must introduce a prepotential P for the conformal compensator. (This prepotential for 4D supergravity was previously introduced in Ref. [21].) We take P to be real and write

$$\Sigma = -\tfrac{1}{4}\bar{D}^2 P, \qquad \delta P = D^\alpha L_\alpha + \text{h.c.} \tag{3.23}$$

We then find that the most general invariant lagrangian that is quadratic in the 5D supergravity fields is

$$\mathcal{L} = \mathcal{L}_{\mathcal{N}=1} + c\Delta\mathcal{L}_5, \tag{3.24}$$

where $\mathcal{L}_{\mathcal{N}=1}$ is the lagrangian of linearized $\mathcal{N} = 1$ supergravity [16]:

$$\begin{aligned}\mathcal{L}_{\mathcal{N}=1} = \int d^4\theta \Big[&\tfrac{1}{8} V^m D^\alpha \bar{D}^2 D_\alpha V_m + \tfrac{1}{48}\left([D^\alpha, \bar{D}^{\dot\alpha}] V_{\alpha\dot\alpha}\right)^2 - (\partial^m V_m)^2 \\ &- \tfrac{1}{3}\Sigma^\dagger\Sigma + \tfrac{2i}{3}(\Sigma - \Sigma^\dagger)\partial^m V_m \Big].\end{aligned} \tag{3.25}$$

and

$$
\begin{aligned}
\Delta\mathcal{L}_5 = \int d^4\theta \Big\{ & \Big[T^\dagger(\Sigma - i\partial_{\alpha\dot\alpha}V^{\dot\alpha\alpha}) + \text{h.c.} \Big] \\
& - \tfrac{1}{2}\Big[D^\alpha \hat\Psi_\alpha + \bar D_{\dot\alpha}\hat\Psi^{\dagger\dot\alpha} - \partial_5 P \Big]^2 \\
& + \Big[\partial_5 V_{\alpha\dot\alpha} - (\bar D_{\dot\alpha}\hat\Psi_\alpha - D_\alpha \hat\Psi^\dagger_{\dot\alpha}) \Big]^2 \Big\},
\end{aligned}
\tag{3.26}
$$

The constant c is to be determined by imposing 5D Lorentz invariance.

If we write the action as $S = M_5^3 \int d^5x \mathcal{L}$, then the superfields have the following mass dimensions:

$$
[V] = -1, \quad [P] = -1, \quad [\hat\Psi] = -\tfrac{1}{2}, \quad [T] = 0. \tag{3.27}
$$

With this convention, propagating bosonic fields have dimension 0.

3.3 Components

We now work out the bosonic part of the lagrangian Eq. (3.24). We define the components by covariant projection, as before. Our definitions are chosen to obtain simple transformation laws.

The bosonic components of T are defined to be

$$
t = T|, \tag{3.28}
$$

$$
F_T = -\tfrac{1}{4} D^2 T|. \tag{3.29}
$$

The bosonic components of $\hat\Psi_\alpha$ are defined to be

$$
u_{\alpha\beta} = D_\alpha \hat\Psi_\beta|, \tag{3.30}
$$

$$
v_{\alpha\dot\alpha} = -2i\bar D_{\dot\alpha}\hat\Psi_\alpha|, \tag{3.31}
$$

$$
w_{\alpha\beta} = -\tfrac{1}{4} D_\alpha \bar D^2 \hat\Psi_\beta|, \tag{3.32}
$$

$$
y_{\alpha\dot\alpha} = -\tfrac{1}{4} D^2 \bar D_{\dot\alpha}\hat\Psi_\alpha|. \tag{3.33}
$$

The bosonic components of $V_{\alpha\dot\alpha}$ are defined as before:

$$
c_{\alpha\dot\alpha} = V_{\alpha\dot\alpha}|, \tag{3.34}
$$

$$
a_{\alpha\dot\alpha} = -\tfrac{1}{4} D^2 V_{\alpha\dot\alpha}|, \tag{3.35}
$$

$$
\tilde h_{\alpha\dot\alpha\beta\dot\beta} = -\tfrac{1}{2}[D_\alpha, \bar D_{\dot\alpha}] V_{\beta\dot\beta}|, \tag{3.36}
$$

$$
d_{\alpha\dot\alpha} = \tfrac{1}{32}\{D^2, \bar D^2\} V_{\alpha\dot\alpha}|. \tag{3.37}
$$

The bosonic components of the prepotential P are

$$\varrho = P|, \tag{3.38}$$

$$\sigma = -\tfrac{1}{4}\bar{D}^2 P|, \tag{3.39}$$

$$\tau_{\alpha\dot{\alpha}} = -\tfrac{1}{2}[D_\alpha, \bar{D}_{\dot{\alpha}}]P|, \tag{3.40}$$

$$D_P = \tfrac{1}{32}\{D^2, \bar{D}^2\}P|. \tag{3.41}$$

The relationship between D_P and F_Σ is

$$F_\Sigma = D_P - \tfrac{i}{2}\partial_m \tau^m. \tag{3.42}$$

We first work out the transformation of these components under the gauge symmetries. We define the bosonic components of the transformation parameter Ω as

$$\omega = \Omega|, \tag{3.43}$$

$$F_\Omega = -\tfrac{1}{4}D^2\Omega|. \tag{3.44}$$

The bosonic components of L_α are

$$\gamma_{\alpha\beta} = D_\alpha L_\beta|, \tag{3.45}$$

$$\xi_{\alpha\dot{\alpha}} = -2i\bar{D}_{\dot{\alpha}} L_\alpha|, \tag{3.46}$$

$$\lambda_{\alpha\beta} = -\tfrac{1}{4}D_\alpha \bar{D}^2 L_\beta|, \tag{3.47}$$

$$\kappa_{\alpha\dot{\alpha}} = \tfrac{i}{2}D^2\bar{D}_{\dot{\alpha}} L_\alpha|. \tag{3.48}$$

The bosonic components of T transform as

$$\delta t = \partial_5 \omega, \tag{3.49}$$

$$\delta F_T = \partial_5 F_\Omega. \tag{3.50}$$

The bosonic components of Ψ_α transform as

$$\delta u_{\alpha\beta} = \partial_5 \gamma_{\alpha\beta} + \tfrac{1}{2}\epsilon_{\alpha\beta} F_\Omega, \tag{3.51}$$

$$\delta v_m = \partial_5 \xi_m + \partial_m \omega, \tag{3.52}$$

$$\delta w_{\alpha\beta} = \partial_5 \lambda_{\alpha\beta}, \tag{3.53}$$

$$\delta y_m = \tfrac{i}{2}\partial_5 \kappa_m + \tfrac{i}{2}\partial_m F_\Omega. \tag{3.54}$$

The bosonic components of $V_{\alpha\dot\alpha}$ transform as

$$\delta c_m = -\operatorname{Im}(\xi_m), \tag{3.55}$$

$$\delta a_m = \tfrac{i}{2}\kappa_m, \tag{3.56}$$

$$\delta\tilde h_{\alpha\dot\alpha\beta\dot\beta} = \partial_{\alpha\dot\alpha}\operatorname{Re}(\xi_{\beta\dot\beta}) - 2(\varepsilon_{\dot\alpha\dot\beta}\lambda_{\alpha\beta} + \varepsilon_{\alpha\beta}\bar\lambda_{\dot\alpha\dot\beta}), \tag{3.57}$$

$$\delta d_m = -\tfrac{1}{2}\partial^n\partial_n\operatorname{Im}(\xi_m) + \Big[\tfrac{i}{4}(\tilde\sigma_m\sigma_n)_{\dot\alpha\dot\beta}\partial^n\bar\lambda^{\dot\alpha\dot\beta} + \text{h.c.}\Big]. \tag{3.58}$$

The bosonic components of P transform as

$$\delta\varrho = \gamma^\alpha{}_\alpha + \text{h.c.}, \tag{3.59}$$

$$\delta\tau_{\alpha\dot\alpha} = -2\operatorname{Im}(\kappa_{\alpha\dot\alpha}) + \Big[i\partial^\beta{}_{\dot\alpha}(\gamma_{\alpha\beta}+\gamma_{\beta\alpha}) + \text{h.c.}\Big], \tag{3.60}$$

$$\delta\sigma = \partial_m\xi^m - \lambda_\alpha{}^\alpha, \tag{3.61}$$

$$\delta D_P = \partial^m\operatorname{Re}(\kappa_m). \tag{3.62}$$

We now work out the bosonic terms in the lagrangian to check 5D global Lorentz invariance and determine the constant c in Eq. (3.24). We use the gauge freedom in $\gamma^\alpha{}_\alpha$, $\lambda^\alpha{}_\alpha$, $\operatorname{Im}(\xi_m)$, κ_m, and F_Ω to go to a Wess–Zumino gauge where

$$\varrho, \sigma, c_m, a_m, u^\alpha{}_\alpha = 0. \tag{3.63}$$

This gauge eliminates all bosonic fields with dimension less than 0, the dimension of propagating bosonic fields in our conventions. The gauge condition $\sigma = 0$ requires a compensating gauge transformation, so that $\tilde h_{mn}$ does not transform like a canonical 5-bein fluctuation in this gauge. We will make contact with more familiar expressions by writing our final results in terms of the canonical field h_{mn}, given by

$$\tilde h_{(mn)} = h_{mn} - \eta_{mn}h^p{}_p. \tag{3.64}$$

The component lagrangian in this gauge is

$$\begin{aligned}\mathcal{L}_{\mathcal{N}=1} = &-\tfrac{1}{2}(\partial_m\tilde h_{np})^2 + \tfrac{1}{2}(\partial^m\tilde h_{mn})^2 + \tfrac{1}{2}(\partial^m\tilde h_{nm})^2\\ &-\tfrac{1}{6}(\Omega_m)^2 + \tfrac{1}{6}(\partial_m\tilde h)^2 + \tfrac{1}{3}\tilde h\partial_m\partial_n\tilde h^{mn}\\ &+\tfrac{2}{3}d^m\Omega_m - \tfrac{1}{3}|F_\Sigma|^2 + \tfrac{4}{3}(d_m)^2,\end{aligned} \tag{3.65}$$

and

$$
\begin{aligned}
\Delta\mathcal{L}_5 = {} & 2\,\mathrm{Re}(F_T)D_P - \mathrm{Im}(F_T)\partial_m\tau^m - 4d^m\partial_m\,\mathrm{Im}(t) - 2\,\mathrm{Re}(t)\partial_m\partial_n\tilde{h}^{mn} \\
& - |\partial_m v^m + w^\alpha{}_\alpha|^2 - 2\left[\mathrm{Re}(y_{\alpha\dot\alpha}) - \left(\tfrac{i}{2}\partial_{\beta\dot\alpha}u^\beta{}_\alpha + \text{h.c.}\right) - \tfrac{1}{4}\partial_5\tau_{\alpha\dot\alpha}\right]^2 \\
& - \mathrm{Im}(v^m)\partial^2\,\mathrm{Im}(v_m) - 4d^m\partial_5\,\mathrm{Im}(v_m) - 4|y_m|^2 \\
& + \tfrac{1}{4}\left[\partial_{\alpha\dot\alpha}\,\mathrm{Re}(v_{\beta\dot\beta}) - \partial_5\tilde{h}_{\alpha\dot\alpha,\beta\dot\beta} - 4\,\mathrm{Re}(\varepsilon_{\dot\alpha\dot\beta})\right]^2 \\
& - \left(iw_{\beta\alpha}\partial^\beta{}_{\dot\alpha}\,\mathrm{Im}(v^{\dot\alpha\alpha}) + \text{h.c.}\right),
\end{aligned} \tag{3.66}
$$

where $\tilde{h} = \tilde{h}^m{}_m$ and

$$\Omega_m = \varepsilon_{mnpq}\partial^n\tilde{h}^{pq}. \tag{3.67}$$

We now discuss the elimination of the auxiliary fields. The equations of motion of the auxiliary fields F_T and D_P set

$$F_T, D_P, \partial^m\tau_m = 0. \tag{3.68}$$

The equations of motion of the auxiliary field y_m (from $\hat{\Psi}_\alpha$) eliminate all dependence on the fields τ_m and $u_{\alpha\beta}$:

$$
\begin{aligned}
\mathcal{L} = c\Big\{ & \mathrm{Re}(y^{\dot\alpha\alpha})\left[(2i\partial_{\beta\dot\alpha}u^\beta{}_\alpha + \text{h.c.}) + \partial_5\tau_{\alpha\dot\alpha}\right] + \tfrac{1}{2}(\mathrm{Im}(y^m))^2\Big\} \\
& + \text{independent of } y.
\end{aligned} \tag{3.69}
$$

This is important because τ_m and $u_{\alpha\beta}$ have dimension 0, the dimension of a propagating bosonic field. The remaining fields of dimension 0 are just enough to parameterize the bosonic component fields of the theory (see §3.1 above).

The only remaining auxiliary fields are d_m and $w_{\alpha\beta}$. Their equations of motion give

$$d_m = \tfrac{3}{2}c\left[\partial_m\,\mathrm{Im}(t) - \partial_5\,\mathrm{Im}(v_m)\right] - \tfrac{1}{4}\Omega_m, \tag{3.70}$$

$$w_{\alpha\beta} = -\tfrac{1}{3}\epsilon_{\alpha\beta}\partial_5\tilde{h} + \tfrac{1}{8}\left[\partial_{(\alpha}{}^{\dot\alpha}\,\mathrm{Re}(v_{\beta)\dot\alpha}) - \partial_5\tilde{h}_{(\alpha}{}^{\dot\alpha}{}_{\beta)\dot\alpha} - i\partial_{(\alpha}{}^{\dot\alpha}\,\mathrm{Im}(v_{\beta)\dot\alpha})\right]. \tag{3.71}$$

Substituting these back into the action results in a large number of cancellations. In particular, terms of the form $\partial_m\,\mathrm{Re}(v^m)\partial_5\tilde{h}$, $\Omega^m\partial_5\,\mathrm{Im}(v_m)$, and $(\Omega_m)^2$ cancel. Additionally, all further dependence on the antisymmetric part of the 5-bein fluctuation

$\tilde{h}_{[mn]}$ cancels and we are left with

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{2}(\partial_m \tilde{h}_{(np)})^2 + (\partial^m \tilde{h}_{(mn)})^2 + \tfrac{1}{6}(\partial_m \tilde{h})^2 + \tfrac{1}{3}\tilde{h}\partial_m\partial_n\tilde{h}^{(mn)} \\ &- \tfrac{1}{3}c(\partial_5\tilde{h})^2 + c(\partial_5\tilde{h}_{(mn)})^2 - 2c\,\mathrm{Re}(t)\partial_m\partial_n\tilde{h}^{(mn)} \\ &+ \tfrac{1}{4}c\left[\partial_m \mathrm{Re}(v_n) - \partial_n \mathrm{Re}(v_m)\right]^2 - 2c\partial_m \mathrm{Re}(v_n)\partial_5\tilde{h}^{(mn)} \\ &+ \tfrac{3}{4}c\left[\partial_m \mathrm{Im}(v_n) - \partial_n \mathrm{Im}(v_m)\right]^2 - 3c^2\left[\partial_m \mathrm{Im}(t) - \partial_5 \mathrm{Im}(v_m)\right]^2 ,\end{aligned} \tag{3.72}$$

The last two terms give the graviphoton gauge kinetic term. 5-dimensional Lorentz invariance of these terms requires

$$c = -\tfrac{1}{2}. \tag{3.73}$$

The lagrangian is now completely fixed, and gives a 5D Lorentz invariant result. It is convenient to express the final result in terms of the fields

$$h_{mn} = \tilde{h}_{(mn)} - \tfrac{1}{3}\eta_{mn}\tilde{h}^p{}_p, \tag{3.74}$$

$$h_{m5} = \tfrac{1}{2}\,\mathrm{Re}(v_m), \tag{3.75}$$

$$h_{55} = \mathrm{Re}(t), \tag{3.76}$$

$$B_m = \sqrt{\tfrac{3}{2}}\,\mathrm{Im}(v_m), \tag{3.77}$$

$$B_5 = \sqrt{\tfrac{3}{2}}\,\mathrm{Im}(t), \tag{3.78}$$

with transformation laws

$$\delta h_{mn} = \tfrac{1}{2}\left(\partial_m\xi_n + \partial_n\xi_m\right), \tag{3.79}$$

$$\delta h_{m5} = \tfrac{1}{2}\left(\partial_m\xi_5 + \partial_5\xi_m\right), \tag{3.80}$$

$$\delta h_{55} = \partial_5\xi_5, \tag{3.81}$$

$$\delta B_m = \partial_m\alpha, \tag{3.82}$$

$$\delta B_5 = \partial_5\alpha, \tag{3.83}$$

where

$$\xi_5 = \mathrm{Re}(\omega), \tag{3.84}$$

$$\alpha = \sqrt{\tfrac{3}{2}}\mathrm{Im}(\omega). \tag{3.85}$$

In terms of these fields the lagrangian is

$$\begin{aligned}\mathcal{L} = -\tfrac{1}{2}(\partial_m h_{np})^2 + (\partial^m h_{mn})^2 + \tfrac{1}{2}(\partial_m h)^2 + h\partial_m\partial_n h^{mn} \\ + \tfrac{1}{2}(\partial_5 h)^2 - \tfrac{1}{2}(\partial_5 h_{mn})^2 + h_{55}\partial_m\partial_n h^{mn} \\ - \tfrac{1}{2}\left[\partial_m h_{n5} - \partial_n h_{m5}\right]^2 + 2\partial_m h_{n5}\partial_5 h^{mn} - 2\partial^m h_{m5}\partial_5 h \\ - \tfrac{1}{4}\left[\partial_m B_n - \partial_n B_m\right]^2 - \tfrac{1}{2}\left[\partial_m B_5 - \partial_5 B_m\right]^2 .\end{aligned} \tag{3.86}$$

This is the correct 5D lagrangian for linearized gravity plus a graviphoton field.

3.4 Fermions and 5D Supersymmetry

We will not carry out the component expansion of the fermions, but we will show that the theory contains all components of the 5D gravitino with the correct transformation law under infinitesimal local 5D supersymmetry. In the 5D theory, the gravitino Ψ_M transforms under local supersymmetry as

$$\delta\Psi_M = \partial_M \varepsilon. \tag{3.87}$$

In the reduction to $\mathcal{N} = 1$ superspace, the gravitino decomposes into the fields $\psi^{(\pm)}_{M\alpha}$, where the $\pm$ refers to the intrinsic parity under $x^5 \mapsto -x^5$.

Propagating fermion fields have mass dimension $+\frac{1}{2}$ in our conventions. The propagating fermion fields are

$$\psi^{(+)}_{m\alpha} = \left[-\tfrac{1}{3}(\sigma_{mn})_\alpha{}^\beta + \tfrac{5}{6}\eta_{mn}\delta_\alpha{}^\beta\right]\psi^n_\beta, \tag{3.88}$$

$$\psi^{(-)}_{m\alpha} = i(\tilde{\sigma}_m)^{\dot\beta\beta} D_\alpha \bar{D}_{\dot\beta}\hat{\Psi}_\beta| - \tfrac{i}{2}(\sigma_m)_\alpha{}^{\dot\beta} D^2 \hat{\bar{\Psi}}_{\dot\beta}|. \tag{3.89}$$

$$\psi^{(+)}_{5\alpha} = -\tfrac{1}{4}\bar{D}^2\hat{\Psi}_\alpha|. \tag{3.90}$$

$$\psi^{(-)}_{5\alpha} = D_\alpha T|, \tag{3.91}$$

where $\psi_{m\alpha}$ is the 'unconventional' gravitino field defined in Eq. (2.31). These transform as

$$\delta\psi^{(\pm)}_{M\alpha} = \partial_M \varepsilon^{(\pm)}_\alpha, \qquad M = 0, \ldots, 3, 5, \tag{3.92}$$

where the transformation parameters are

$$\varepsilon^{(+)}_\alpha = -\tfrac{1}{4}\bar{D}^2 L_\alpha|, \tag{3.93}$$

$$\varepsilon^{(-)}_\alpha = D_\alpha \Omega|. \tag{3.94}$$

4 Applications

4.1 S^1/Z_2 Orbifold

We now consider the compactification of this theory on an S^1/Z_2 orbifold. This is a good starting point for constructing realistic 'brane world' scenarios, since this compactification breaks supersymmetry down to $\mathcal{N} = 1$ and gives rise to two 'branes' at the orbifold fixed points. In the present formalism, the Z_2 parity assignments are simply

$$\mathcal{P}(V_m) = +1, \qquad \mathcal{P}(P) = +1, \qquad \mathcal{P}(\Psi_\alpha) = -1, \qquad \mathcal{P}(T) = +1. \tag{4.1}$$

It is now simple to couple the 5D supergravity multiplet to matter fields localized on the boundaries, since the 5D supergravity multiplet induces a $\mathcal{N} = 1$ minimal supergravity multiplet on the boundaries at $x^5 = 0, \pi r$. Note that the radion superfield transforms as

$$\delta T = \partial_5 \Omega, \tag{4.2}$$

and $\partial_5 \Omega$ is a general chiral superfield on the boundary. Since superfields localized on the boundary do not transform under Ω, we see that there are no couplings of the radion superfields to the boundary. (This result was obtained in Ref. [6] by an indirect argument.)

Since we have derived the linearized theory, our results apply directly to supergravity couplings to the boundary at linear order. However, it is clear that the fully nonlinear form of these couplings is simply obtained by including the standard nonlinearization of the induced $\mathcal{N} = 1$ supergravity multiplet. Of particular importance is the conformal compensator, given by

$$\phi = e^{\Sigma/3}. \tag{4.3}$$

We can now use the usual nonlinear couplings of $\mathcal{N} = 1$ supergravity to 4D matter. For example, the couplings of brane localized fields to the conformal compensator are given by

$$\delta \mathcal{L}_5 = \delta(x^5) \left[\int d^4\theta \, \phi^\dagger \phi f + \left(\int d^2\theta \, \phi^3 W + \text{h.c.} \right) \right] + \cdots \tag{4.4}$$

where f is the Kähler function and W is the superpotential, and we omit the dependence on V_m.

As an application of this formalism, we show that couplings of 5D supergravity to branes can naturally generate a μ term of realistic size in the context of gaugino mediation [13]. This can be viewed as a 5D version of the Giudice–Masiero mechanism [20].

We consider a theory with standard model Higgs fields $H_{u,d}$ localized on the boundary at $x^5 = 0$. Consider the following brane-localized term added to the 5-dimensional lagrangian:

$$\Delta\mathcal{L}_5 = \delta(x^5) \int d^4\theta \, \phi^\dagger \phi \left[H_u^\dagger H_u + H_d^\dagger H_d + (c H_u H_d + \text{h.c.}) + \cdots \right]. \tag{4.5}$$

where c is a dimensionless coupling. We have omitted the dependence on the supergravity field V_m but we have given the full dependence on the conformal compensator ϕ. Supersymmetry breaking gives rise to $\langle F_\phi \rangle \neq 0$ and generates effective μ and $B\mu$ terms

$$\mu = c \langle F_\phi^\dagger \rangle, \qquad B\mu = -c |\langle F_\phi \rangle|^2. \tag{4.6}$$

In gaugino mediation, supersymmetry breaking is communicated to the bulk gaugino via boundary couplings of the form

$$\Delta\mathcal{L}_5 = \delta(x^5 - \pi r) \int d^2\theta \, \frac{X}{M} W^\alpha W_\alpha + \text{h.c.} + \cdots, \tag{4.7}$$

where X is a chiral superfield whose F component breaks supersymmetry, and W_α is the field strength of the bulk gauge multiplet. To estimate the size of M we assume that at the cutoff of the theory Λ where all loop effects are suppressed by a factor of $\epsilon \lesssim 1$ [22]. This gives

$$\Lambda \sim M_4, \qquad 2\pi r \sim \frac{24\pi^3 \epsilon}{M_4}, \tag{4.8}$$

and

$$c \sim 1, \qquad M \sim 4\pi\sqrt{\epsilon} M_4. \tag{4.9}$$

This implies that $B \sim \mu$, with

$$\frac{\mu}{m_{1/2}} \sim \frac{1}{4\pi\sqrt{\epsilon}}. \tag{4.10}$$

This gives μ and $B\mu$ terms of the right order of magnitude for $10^{-2} \lesssim \epsilon \lesssim 1$. Sequestering requires $2\pi r \Lambda \sim 24\pi^3 \epsilon \gtrsim 7$, which is satisfied for all ϵ in this range.

Similar results hold for radion-mediated supersymmetry breaking (third paper in Ref. [4]). Eqs. (4.8) and (4.9) are still valid, but $m_{3/2} \sim \langle F_T/T \rangle \sim \frac{1}{10}$. This gives $B \sim \mu$ with

$$\frac{\mu}{m_{1/2}} \sim 10. \tag{4.11}$$

This may be acceptable given the uncertainties in the estimates above.

5 Conclusions

To summarize, we have presented an embedding of linearized 5D supergravity into $\mathcal{N} = 1$ superfields, that is functions of 4D superspace (x^m, θ_α), together with x^5. The propagating fields are embedded in superfields as follows. The fields h_{mn} and $\psi^{(+)}_{m\alpha}$ are embedded in the standard $\mathcal{N} = 1$ supergravity superfield

$$V_m \sim \theta\sigma^n\bar{\theta}h_{mn} + \bar{\theta}^2\psi^{(+)}_{m\alpha} + \cdots = \text{real}, \tag{5.1}$$

and the remaining propagating fields are embedded into the superfields

$$\Psi_\alpha \sim \bar{\theta}^{\dot{\alpha}}\sigma^m_{\alpha\dot{\alpha}}(B_m + ih_{5m}) + \theta\sigma^m\bar{\theta}\psi^{(-)}_{m\alpha} + \bar{\theta}^2\psi^{(+)}_{5\alpha} + \cdots = \text{unconstrained}, \tag{5.2}$$

$$T \sim h_{55} + iB_5 + \theta^\alpha\psi^{(-)}_{5\alpha} + \cdots = \text{chiral}, \tag{5.3}$$

in a gauge where $h_{5m} = h_{m5}$. Additionally, the theory contains a real superfield P that acts as a prepotential for the conformal compensator. On an S^1/Z_2 orbifold, V_m, P, and T are even, while Ψ_α is odd. The 5D lagrangian is given in Eqs. (3.25) and (3.26). The induced supergravity multiplet on the boundary is the usual $\mathcal{N} = 1$ supergravity multiplet, so coupling to boundary supermultiplets is simple.

We believe that this formalism will be useful in systematically including supergravity effects in higher-dimensional theories and 'brane-world' scenarios. As a first step in this direction, we have shown how couplings of 5D supergravity to boundary Higgs fields can give realistic μ and $B\mu$ terms in the context of gaugino- and radion-mediated supersymmetry breaking. There are numerous open directions for future work. These include the generalization to other backgrounds (such as 'warped' compactifications), the extension beyond linear order, coupling to bulk hypermultiplets and gauge multiplets, and generalizations to dimensions higher than 5.

Acknowledgements

We thank S. James Gates, Jr. and R. Sundrum for useful discussions. W.D.L. and J.P. were supported by the University of Maryland Center for String and Particle Theory. M.A.L. was supported by NSF grant PHY-0099544.

References

[1] A. H. Chamseddine, R. Arnowitt and P. Nath, "Locally supersymmetric grand unification," Phys. Rev. Lett. **49**, 970 (1982); R. Barbieri, S. Ferrara and C. A. Savoy, "Gauge models with spontaneously broken local supersymmetry," Phys. Lett. B **119**, 343 (1982); L. J. Hall, J. Lykken and S. Weinberg, "Supergravity as the messenger of supersymmetry breaking," Phys. Rev. D **27**, 2359 (1983).

[2] P. Horava and E. Witten, "Eleven-dimensional supergravity on a manifold with boundary," Nucl. Phys. B **475**, 94 (1996) [arXiv:hep-th/9603142].

[3] L. Randall and R. Sundrum, "Out of this world supersymmetry breaking," Nucl. Phys. B **557**, 79 (1999) [arXiv:hep-th/9810155].

[4] I. Antoniadis, "A possible new dimension at a few Tev," Phys. Lett. B **246**, 377 (1990); G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, "Gaugino mass without singlets," JHEP **9812**, 027 (1998) [arXiv:hep-ph/9810442]; Z. Chacko and M. A. Luty, "Radion mediated supersymmetry breaking," JHEP **0105**, 067 (2001) [arXiv:hep-ph/0008103]; M. A. Luty, "Weak scale supersymmetry without weak scale supergravity," arXiv:hep-th/0205077.

[5] H. P. Nilles, M. Olechowski and M. Yamaguchi, "Supersymmetry breakdown at a hidden wall," Nucl. Phys. B **530**, 43 (1998) [arXiv:hep-th/9801030]; K. A. Meissner, H. P. Nilles and M. Olechowski, "Supersymmetry breakdown at distant branes: The super-Higgs mechanism," Nucl. Phys. B **561**, 30 (1999) [arXiv:hep-th/9905139]; J. R. Ellis, Z. Lalak, S. Pokorski and S. Thomas, "Supergravity and supersymmetry breaking in four and five dimensions," Nucl. Phys. B **563**, 107 (1999) [arXiv:hep-th/9906148]; J. Bagger, F. Feruglio and F. Zwirner, "Brane induced supersymmetry breaking," JHEP **0202**, 010 (2002) [arXiv:hep-th/0108010]; K. A. Meissner, H. P. Nilles and M. Olechowski, "Brane induced supersymmetry breakdown and restoration," arXiv:hep-th/0205166.

[6] M. A. Luty and R. Sundrum, "Radius stabilization and anomaly-mediated supersymmetry breaking," Phys. Rev. D **62**, 035008 (2000) [arXiv:hep-th/9910202]; "Hierarchy stabilization in warped supersymmetry," Phys. Rev. D **64**, 065012 (2001) [arXiv:hep-th/0012158].

[7] T. Gherghetta and A. Riotto, "Gravity-mediated supersymmetry breaking in the brane-world," Nucl. Phys. B **623**, 97 (2002) [arXiv:hep-th/0110022].

[8] M. Zucker, "Minimal off-shell supergravity in five dimensions," Nucl. Phys. B **570**, 267 (2000) [arXiv:hep-th/9907082].

[9] A. Ceresole and G. Dall'Agata, "General matter coupled $\mathcal{N} = 2$, $D = 5$ gauged supergravity," Nucl. Phys. B **585**, 143 (2000) [arXiv:hep-th/0004111]; T. Kugo and K. Ohashi, "Supergravity tensor calculus in 5D from 6D," Prog. Theor. Phys. **104**, 835 (2000) [arXiv:hep-ph/0006231]; M. Zucker, "Supersymmetric brane world scenarios from off-shell supergravity," Phys. Rev. D **64**, 024024 (2001) [arXiv:hep-th/0009083]; T. Fujita, T. Kugo and K. Ohashi, "Off-shell formulation of supergravity on orbifold," Prog. Theor. Phys. **106**, 671 (2001) [arXiv:hep-th/0106051]; T. Kugo and K. Ohashi, "Superconformal tensor calculus on orbifold in 5D," arXiv:hep-th/0203276.

[10] N. Arkani-Hamed, L. J. Hall, D. R. Smith and N. Weiner, "Exponentially small supersymmetry breaking from extra dimensions," Phys. Rev. D **63**, 056003 (2001) [arXiv:hep-ph/9911421]; N. Arkani-Hamed, T. Gregoire and J. Wacker, "Higher dimensional supersymmetry in 4D superspace," JHEP **0203**, 055 (2002) [arXiv:hep-th/0101233].

[11] N. Marcus, A. Sagnotti and W. Siegel, "Ten-dimensional supersymmetric Yang-Mills theory in terms of four-dimensional superfields," Nucl. Phys. B **224**, 159 (1983).

[12] D. Marti and A. Pomarol, "Supersymmetric theories with compact extra dimensions in $\mathcal{N} = 1$ superfields," Phys. Rev. D **64**, 105025 (2001) [arXiv:hep-th/0106256]; D. E. Kaplan and N. Weiner, "Radion mediated supersymmetry breaking as a Scherk-Schwarz theory," arXiv:hep-ph/0108001.

[13] D. E. Kaplan, G. D. Kribs and M. Schmaltz, "Supersymmetry breaking through transparent extra dimensions," Phys. Rev. D **62**, 035010 (2000) [arXiv:hep-ph/9911293]; Z. Chacko, M. A. Luty, A. E. Nelson and E. Ponton, "Gaugino mediated supersymmetry breaking," JHEP **0001**, 003 (2000) [arXiv:hep-ph/9911323].

[14] W. Siegel and S. J. Gates, "Superfield supergravity," Nucl. Phys. B **147**, 77 (1979).

[15] S. J. Gates, M. T. Grisaru, M. Rocek and W. Siegel, "Superspace, or One Thousand and One Lessons in Supersymmetry," Front. Phys. **58**, 1 (1983) [arXiv:hep-th/0108200];

[16] I. L. Buchbinder and S. M. Kuzenko, "Ideas and Methods of Supersymmetry and Supergravity: A Walk Through Superspace," *IOP (1995)*.

[17] J. Wess and J. Bagger, "Supersymmetry and Supergravity," *Princeton University Press (1990)*.

[18] E. Cremmer, in "Superspace and Supergravity", Eds. S. W. Hawking and M. Rocek *Cambridge University Press, 1981*, pp. 267-282.

[19] S. J. Gates and W. Siegel, "$(\frac{3}{2}, 1)$ superfield of $O(2)$ supergravity," Nucl. Phys. B **164**, 484 (1980).

[20] G. F. Giudice and A. Masiero, "A natural solution to the μ problem in supergravity theories," Phys. Lett. B **206**, 480 (1988).

[21] S. J. Gates, "Super p form gauge superfields," Nucl. Phys. B **184**, 381 (1981); S. J. Gates and W. Siegel, "Variant superfield representations," Nucl. Phys. B **187**, 389 (1981).

[22] Z. Chacko, M. A. Luty and E. Ponton, "Massive higher-dimensional gauge fields as messengers of supersymmetry breaking," JHEP **0007**, 036 (2000) [arXiv:hep-ph/9909248].